Mass-Spring Parameters Definition in 2D for Simulation

TROUVAIN G. [1][2]
PHD student
guillaume.trouvain@ifma.fr

GAGNOL V. [1]
Assistant Professor
vincent.gagnol@ifma.fr

DUC E. [1]
Professor
emmanuel.duc@ifma.fr

SANCHO J.F. [2]
Michelin Engineer
jean-francois.sancho@fr.michelin.com

[1] Clermont Université, Institut Pascal, UBP-CNRS-IFMA, Campus des Cézeaux
24 Avenue des Landais, 63177 AUBIERE - FRANCE

[2] Centre de Technologies Michelin, ZI de Ladoux, 63118 CEBAZAT - FRANCE

ABSTRACT

In computer graphics and in industrial context, Mass-Spring model is used to obtain fast and visual results in physical simulations. A disadvantage of the method is to obtain accurate result on account of the difficulty to define parameters of a Mass-Spring Model. Different works and results are carried out to define Mass-Spring parameters in other domains such as in cloth animation or in soft tissue modeling. However the Mass-Spring model is not used in some context where a real-time computation can be useful as in tire manufacturing industry for example. In this paper, a method is presented to define the geometric configuration of a Mass-Spring system and the tuning of the mass, stiffness and damper parameters according to physical material behaviours. Different load cases are studied and used to conduct a sensitivity study on the network spring parameters. Then, results are compared to Finite Element Model of same cases in order to evaluate the precision of the proposed approach.

Keywords

Physically-based simulation, Mass-Spring System, Mechanical behavior, spring stiffness

1 INTRODUCTION

Concerning the deformation methods, two habitual approaches are documented in the bibliography. First, Geometrically-based methods such as the Free Form Deformation (FFD) can be used to model deformation of a part. However these methods do not take into account physical parameters and results are not enough accurate to model the physical reality [Sed86]. Second, Physical-based methods take into account physical parameters. These methods can be used to model the deformation of a part integrating material characteristics in a mechanical simulation. In this category, Finite Element Method [Bat96], Boundary Element Method [Jam99] [Tan06], Tensor Element Method [Cot99] [Pic03] and Mass-Spring model are the most known.

Mass Spring method is widely developed to obtain fast and accurate results in the textile and surgery domain. Nevertheless, works about the Mass-Spring Method show that deformation results depend on the quality of the parameters definition. This article proposes an approach to define Mass-Spring system configuration and parameters for steel and rubber materials. The approach is developed to compute the shape deformations in industrial context.

In industrial context, accuracy as well as computation time are important criteria to validate results of a deformation method. FEM results can be very accurate to model physical reality. However, the computation time is too long to be acceptable for "real time simulation" inferior to a few seconds, especially in the case where a large number of boundary conditions should be applied. Note that this feature is particularly important in the case of the treated industrial problem about contact between different parts. In this way, to reduce the computation time of FEM, some methods based on a pre-computation are developed for Finite Element Method and Tensor Element Method [Pac05]. The pre-computation enables to build a database to compute a body deformation in several same models where boundary conditions can be modified. The computation of the database costs more time of the deformation simulation. Nevertheless, a pre-computation must be done for each model configuration. This disadvantage is not acceptable in an industrial context when a real-time computation is required.

Thus, for the simulation of deformable piece according to industrial process, the computation time of Mass-Spring model can be very interesting although the accuracy is known to be lower than other physical-based methods. The deformation accuracy modeled by a Mass-Spring depends of the Mass-Spring system geometry as well as the tuning of system parameters. It includes masses of each particle, the stiffness of each spring and the distribution of springs in the model. Different methods to define the parameters of a Mass-Spring system are developed and presented in the bibliography section of this article as [Bia04], [Pac05] and [Bau09] for example. The methods concern particularly soft-tissue deformation and textile modeling whereas Mass-Spring model can be useful in other domains such as in tire manufacturing industry too. Thus an approach to define the Mass-Spring system parameters is presented in this article for a special distribution method of the springs. The interest of our approach is to model flexible materials such as rubber as very rigid materials such as steel.

The rest of the paper is organized as follows. In the next section, some previous works dealing with the definition of Mass-Spring system parameters are presented. In the third section, our proposed method to characterize mechanical system is detailed. In the fourth section, several examples are presented to illustrate the proposed method. Finally, conclusions and perspectives are given in last section.

2 RELATED WORKS

Physically-based methods were described for the first time by Terzepoulos et al [Ter88] [Ter87]. These methods are used to represent physical phenomena with different levels of rapidity and accuracy. Since these initial works, other methods have been used to model deformable objects and have been described in [Nea05].

The Mass-Spring model is widely used in computer graphics context and particularly to model soft tissue, such as deformation of organ in the surgery domain [Tan09] or in textile deformation [Pro96]. Therefore, Mass-Spring model can be used in the animation context [Mil88]. The method presents a simple structure which enables to model large deformation. Indeed, the method is based on a mesh of a part where nodes are particles and edges are springs. Each particle owns a mass and each spring owns a stiffness and a damping which should be characterized according to the parts of physical characteristics [Jar13].

About the mass distribution of a modeled object, the used method should enable to divide the whole mass to each particle. The discretization is able to obtain the same inertia center than in the physics reality for an object. In this way, the Equation 1 is defined in 2D domain for a quadrangle mesh. In this equation, m_i is the mass of the particle i, ρ the mass density and A_{ej} the area of each element which contains the particle i (Figure 1). This definition is commonly used in different works such as in [Pac05] [Che07] and it is also used in the proposed approach.

$$m_i = \sum_{j=1}^{n} \left(\frac{\rho A_{ej}}{4}\right) \tag{1}$$

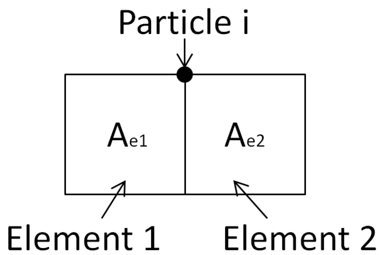

Figure 1: Example to compute mass of particle

Concerning the rigidity of each spring, the definition is an important problem and represents a current research topic. Indeed the influence of this parameter is important on the accuracy of the results since springs enable to model the stiffness of the materials. To realize this definition, different types of methods are proposed in the literature.

First, definition of the spring stiffness can be done using simulated annealing algorithms or genetic algorithms. The algorithms give access to spring stiffness constants. The principle of this method is based on the application of random values to different springs constants and by comparing the obtained model with some mechanical experiments in which results are either well known analytically or can be obtained by Finite Element Methods. However, in this definition, the characterization process must be done for each mesh configuration [Bia03] [Bia04].

Second, another approach is developed by Van Gelder to compute each spring stiffness according to material parameters such as Young's modulus and Poisson's ratio, of the deformed object [Van98]. Van Gelder's approach is used with triangular meshes to simulate hyper-elastic behavior with small or large deformation. Therefore, the method is referenced and used in different works [Che07]. Baudet shows that Van Gelder's approach is not valid to compute mechanical deformation according to the physical reality. His demonstration is based on a Lagrangian analysis and numerical simulations [Bau06].

An approach based on the comparison of stiffness matrix computed from the studied system between Mass-Spring model and FEM is developed by Lloyd [Llo07]. His approach is based on several steps. First, from FEM the stiffness matrix is analytically computed. Then, an element for the Mass-Spring model corresponds to an element used in the FEM is chosen. Third, equations of the Mass-Spring model are linearized and are used to identify the stiffness of each spring from the stiffness matrix of the FEM. This approach is based on analytically computation to define parameters, nevertheless an FEM method close to the Mass-Spring model is required to make the computations.

In this way, another method based on an analytical theory is developed by Baudet [Bau06]. The approach is developed in 2D domain then extended in 3D domain. In 2D domain, each spring stiffness constant is computed according to Young's modulus and Poisson's ratio such as the Van Gelder's approach. The study is done from a quadrangle element which contains three pairs of springs that stiffness constant must be computed for one element (Figure 2 where K_d, K_{ho}, K_{lo} are the spring stiffness). Therefore, to model complex form, elements are assembled and the spring stiffness of adjacent elements are add up to find the equivalent spring stiffness in the system (Figure 3). This method is improved by Flechon who adds correction forces to model the incompressibility behaviour of materials [Fle11]. Nevertheless this method is only validated for soft tissue modeling.

Note that elements of mesh such as element of the Figure 2 are not usually used in mechanical simulation.

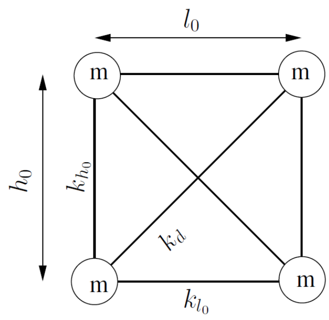

Figure 2: 2D Mesh element proposed by Baudet [Bau09]

The presented works in this section introduce the definition of parameters in a Mass-Spring system. However, approaches are validated to model the behaviour of materials with low Young modulus inferior to 1 KPa.

In this way, materials such as rubber and metal are studied in this paper to show the possibility to use the Mass-Spring model in other industrial contexts where computation time is suitable for real-time applications.

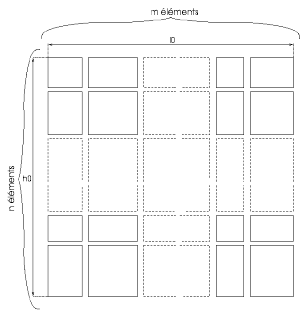

Figure 3: System of n*m elements [Bau06]

3 PROPOSED METHOD

The goal of the proposed approach is to determinate the spring stiffness of a Mass-Spring system to model materials such as rubber and steel. These materials have a Young modulus varying between 1MPa and 210000MPa. The behaviour of the studied materials is considered linear and isotropic. These hypotheses allow to apply the same stiffness to all the springs.

In the related works, the mesh system depends on the mechanical behaviour of the material. Indeed a regular mesh elements such as in the Figure 2 is used to model an isotropic behaviour (Figure 4a) whereas another regular mesh (Figure 4b) is used to model an anisotropic behaviour. The both meshes of the Figure 4 require to create a structured mesh to model the material behaviour. In this way, accuracy of the deformation method depends on the quality of the mesh. In the proposed approach, a meshing based on the Delaunay triangulation is created. Indeed the proposed approach allows to compute meshing without building constraints. This hypothesis allows to use any type of mesh equivalent to FEM mesh.

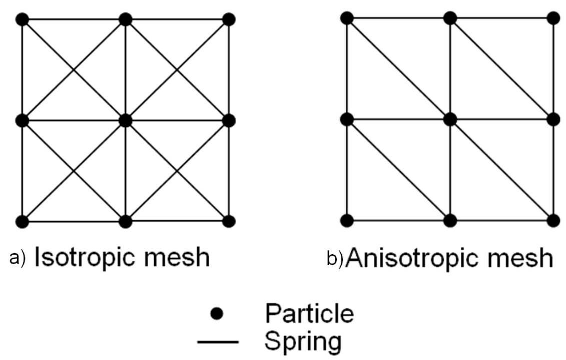

Figure 4: Meshs used to Mass-Spring model

Specifically, to define of Mass-Spring system parameters, two meshes based on the Delaunay triangulation are used. In a case, called "free meshing", the size of the springs on the edge of the part is imposed, then the mesh is free (Figure 5). In the other case, called "structured meshing", the size of each spring is imposed in the whole meshing (Figure 6). Tests of traction and compression are applied on the two different meshes with three different sizes of beam.

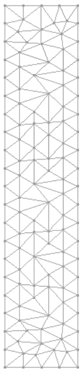

Figure 5: Free Delaunay Triangulation, "free meshing"

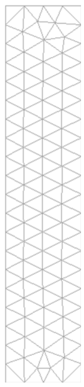

Figure 6: Structured Delaunay Triangulation, "structured meshing"

First, a sensitivity study is conducted for the size beam 200*30 to define stiffness which should be applied to model steel material behaviour. This study is done for the both mesh "structured meshing" and "free meshing" in the case of traction test. For the first mesh type, results show that the deformation error is minimum with a stiffness of $170E6\ N/m$ (Figure 7). For the second mesh type, results are not shown in this article but the deformation error is minimum with a stiffness of $185E6\ N/m$ (Figure 8). The results show that the spring stiffness depends on the topology of the used mesh.

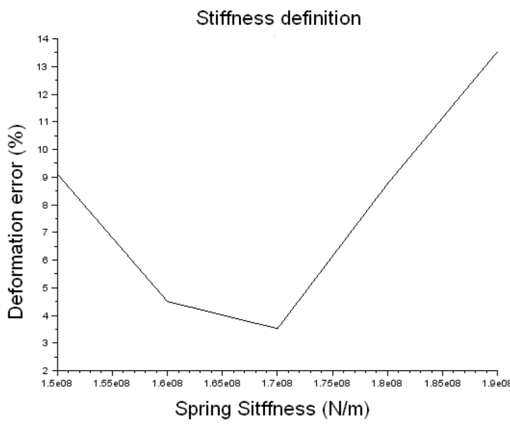

Figure 7: Sensitivity study to define spring stiffness to model steel material behaviour
(beam of 200*30 and "structured meshing")

Then, another sensitivity studies are conducted with the traction and compression tests about the size of meshing and the size of the part. Indeed these parameters can have an influence on the parameters definition. The studies are done in each case with the stiffness defined with the first sensitivity study. Results are shown in the Tables 1, 2, 3 and 4. In each Table, the deformation error is computed about the difference of geometrical deformation computed comparing the Mass-Spring Model to FEM model.

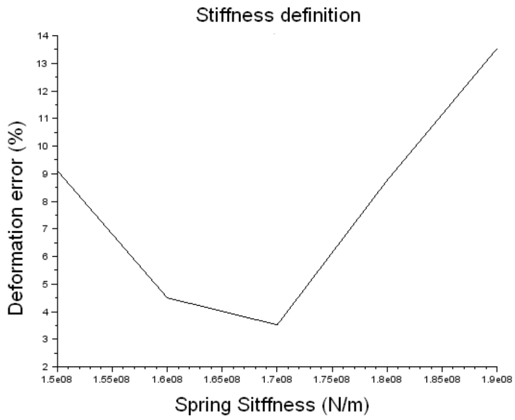

Figure 8: Sensitivity study to define spring stiffness to model steel material behaviour
(beam of 200*30 and "free meshing")

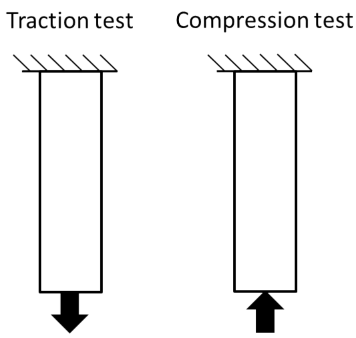

Figure 9: Tests of traction and compression done in Mass-Spring model compared to FEM

Note that a single value is given for the FEM deformation since an adapted mesh is used for this test. Moreover the sizes of beams are chosen arbitrary for this first study to model steel material behaviour.

According to the results (Tables 1, 2, 3 and 4), stiffness of springs does not depend on the size of the beam and on the size of element for each type of meshing. However for an element size equal to 1mm, computation is not always more accurate than a computation with an element size equal to $5mm$. Note that is not the case comparing to the FEM for which a finer mesh gives better results than a coarse mesh.

In the cases where the deformation error is superior to 10%, the geometrical error is inferior to 1mm. However, it is important to note that the proposed approach

Beam Size	200*40			200*30			100*20		
Material	Steel E=210000MPa								
Applied Force (N)	400000			300000			200000		
Size triangular Mesh (mm)	1	2	5	1	2	5	1	2	5
FEM deformation (mm)	9.49			9.5			0.475		
Mass-Spring deformation (mm)	9.57	8.68	9.32	8.99	9.16	8.74	0.45	0.44	0.47
Stiffness of springs	**170E6 N/m**								
Deformation Error (%)	0.78	8.61	1.84	7.95	3.52	5.34	3.79	6.68	0.38

Table 1: Comparison between Mass-Spring model and FEM for **traction test (structured meshing)**

Beam Size	200*40			200*30			100*20		
Material	Steel E=210000MPa								
Applied Force (N)	400000			300000			200000		
Size triangular Mesh (mm)	1	2	5	1	2	5	1	2	5
FEM deformation (mm)	9.49			9.5			0.475		
Mass-Spring deformation (mm)	10.04	9.58	8.74	9.97	9.81	8.13	0.44	0.55	0.39
Stiffness of springs	**185E6 N/m**								
Deformation Error (%)	5.67	0.92	7.95	4.96	3.30	**14.41**	7.35	**16.45**	**17.67**

Table 2: Comparison between Mass-Spring model and FEM for **traction test (free meshing)**

Beam Size	200*40			200*30			100*20		
Material	Steel E=210000MPa								
Applied Force (N)	40000			30000			20000		
Size triangular Mesh (mm)	1	2	5	1	2	5	1	2	5
FEM deformation (mm)	0.95			0.95			0.475		
Mass-Spring deformation (mm)	0.97	0.88	0.93	0.88	0.93	0.90	0.45	0.45	0.48
Stiffness of springs	**170E6 N/m**								
Deformation Error (%)	2.48	6.57	5.24	5.30	1.66	7.26	6.04	5.37	2.40

Table 3: Comparison between Mass-Spring model and FEM for **compression test (structured meshing)**

Beam Size	200*40			200*30			100*20		
Material	Steel E=210000MPa								
Applied Force (N)	40000			30000			20000		
Size triangular Mesh (mm)	1	2	5	1	2	5	1	2	5
FEM deformation (mm)	0.95			0.95			0.475		
Mass-Spring deformation (mm)	1.05	0.98	0.91	1.02	1.05	0.83	0.45	0.57	0.41
Stiffness of springs	**185E6 N/m**								
Deformation Error (%)	**10.96**	3.38	7.74	7.06	9.38	**12.73**	6.04	**21.12**	**12.68**

Table 4: Comparison between Mass-Spring model and FEM for **compression test (free meshing)**

is used in CAD domain. Thus, applications errors are eligible to evaluate the deformation of the studied part.

In this section, the approach is developed for only a steel material with two different mechanical loading cases. Thus, in the next section, applications are done to test the approach with different mechanical cases.

Note that for all the other tests done in this article a "structured meshing" is used.

4 RESULTS

4.1 A compression test with another material

In this section, Mass-Spring model is applied a mechanical load case with rubber material (Young's Modulus 1MPa and Poisson's ratio 0.5). The studied case is a compression test done with a beam of 200mm*40mm. A structured meshing is used for the Mass-Spring Model (Figure 6). Concerning the loading, 0.1MPa pressure is applied to the large surface of the beam and the other large surface is fixed (Figure 10).

Before doing the test, as in the previous case, a sensitivity study is done to define the stiffness of spring which is applied to model rubber material behaviour. Then same traction and compression tests realized in the previous section (Figure 9) are done to define parameters (Table 5) for a rubber material (Young's Modulus 1MPa and Poisson's ratio 0.5). Results give a springs stiffness of $800N/m$ in the case of a structured mesh (Figure 6).

Beam Size	200*40			200*30			100*20		
Material	Steel E=1MPa								
Applied Force (N)	0.4			0.3			0.2		
Size triangular Mesh (mm)	1	2	5	1	2	5	1	2	5
FEM deformation (mm)	1.98			1.98			0.99		
Mass-Spring deformation (mm)	2.10	1.90	2.00	1.90	2.00	1.90	0.90	0.90	1.00
Stiffness of springs	**800 N/m**								
Deformation Error (%)	5.74	4.33	0.70	4.47	0.55	4.47	9.27	9.27	0.81

Table 5: Comparison between Mass-Spring model and FEM for **traction test (structured meshing)**

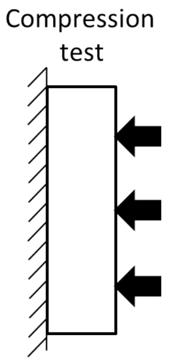

Figure 10: Tests done in Mass-Spring model compared to FEM

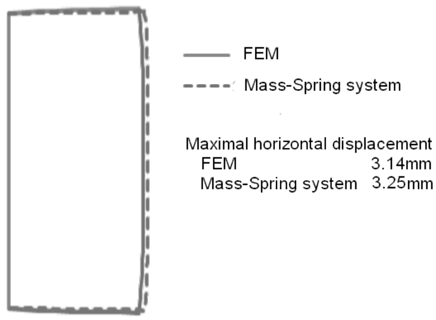

Figure 11: Comparison between Mass-Spring system and FEM results with rubber material for the compression test

To compare the Mass-Spring method to FEM results, visual results are shown in the Figure 11. However test done with this stiffness of $800N/m$ gives a correct visual results but compared to FEM, the minimal horizontal displacement is equal to $4.1mm$ instead of $3.14mm$. This difference of displacement represents an error of 27%.

In this way, same test is done with a stiffness determined empirically of $1000N/m$ applied to each spring. Results are given in the Figure 11, the minimal horizontal displacement is equal to $3.25mm$ for the Mass-Spring model and to $3.14mm$ for FEM. This difference represents an error of 6%.

To conclude, results of the tests done with rubber material show that the defined spring stiffness depends on the loading too. Indeed, for a given mesh, a structured meshing in this test, the defined stiffness with the test done in the previous section 3 is too low. In the case where the loading is done of the large surface of the studied beam, the spring stiffness must be upper. Nevertheless, the defined stiffness of the section 3 gives a correct and fast visual deformation usable in CAD domain.

4.2 Laying of part on another part

The last example allow to use results of the previous sections with a specific mechanical test from the process of tire manufacturing in the industrial context. For this test (Figure12), a rectangular shape composed of rubber material (Young's Modulus 1MPa) is laying on a trapezoidal shape composed of steel material (Young's Modulus 210000MPa). A method based on a geometrical computation is used to detect collision when the rectangular shape is laid on the trapezoidal shape. A structured meshing of the both part is used for the computation of the deformation. From results of the previous application, stiffness of springs depends on the loading and on the mesh. In this way, for this new mesh, a sensitivity study is done to determine the spring stiffness of the rectangular shape. Thus, the stiffness used is $6000N/m$ for the rectangular shape to model rubber material and $170E6\ N/mm$ for the trapezoidal shape to model steel material. Concerning the mesh of the both geometrical shapes, a structured mesh with an imposed size of $1mm$ for the edge is used.

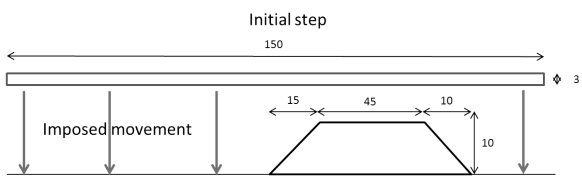

Figure 12: Laying of a part on another part

To compare the Mass-Spring method and the FEM, visual results are shown on the Figure 13 for the rectangular shape. Results are separated to compare the behaviour difference between the rectangular shape and the trapezoidal shape. Concerning the rectangular shape, visual results show that the stiffness applied on springs of the Mass-Spring model is valid for this mechanical case. Concerning the trapezoidal shape, visual results show that the deformed part is closed to the physical reality and the difference of the deformation of this shape is very low for the Mass-Spring model compared the FEM.

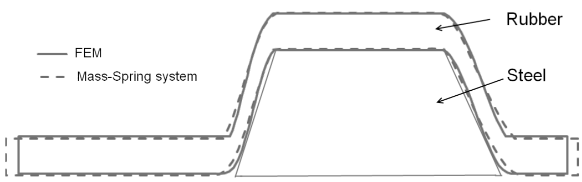

Figure 13: Comparison between FEM and Mass-Spring model for the rectangular shape

To compare the Mass-Spring method and the FEM, the area under the rubber material is computed and it is compared to the area of the trapezoidal shape. The area of the initial trapezoidal shape is $585mm^2$. The hypothesis of full contact between the both shapes is done. Thus the area under the rectangular shape should be equal to the area of the trapezoidal shape. However, for the Mass-Spring model, the area is equal to $600.04mm^2$ and for the FEM, $585.62mm^2$. In the case of the Mass-Spring model the error is 2.65% and for the FEM the error is 0.15%. For the treated industrial problem, a model is considered accurate is the error computed between theoretical model (the reference model) and the other models (Mass-Spring model and FEM) is inferior to 5%. So, in this case, the behaviour of the Mass-Spring model is comparable to FEM and the Mass-Spring model is accurate.

Another comparison is done between the elongation of the rectangular shape at the end of the simulation and the length initial of the shape. Thus, for the Mass-Spring model, the rectangular shape undergoes an elongation of 4.17% and for the FEM 0.54%. So the rectangular shape undergoes an elongation more important with the Mass-Spring model than with the FEM but the Mass-Spring model is accurate to model the treated problem.

To conclude, Results of this test show that Mass-Spring model can be used to compute geometrical deformation between two polygonal shapes. Moreover results of the applications done with the stiffness computed with the approach developed in the previous section are not close to the physical reality, however visual geometrical results are coherent.

Note that in this article, all tests are carried out on a computer equipped with a 2.67 GHz processor, 3.00 GB of RAM and Microsoft Windows XP. The Mass-Spring tests are performed with the platform SOFA developed by the INRIA [All07] and the FEM tests are performed with the software Abaqus V6.12.

5 CONCLUSION

An approach to define Mass-Spring system parameters is proposed for the simulation of body deformation in 2D domain. Different meshing, loading, and materials are studied to determine the sensitivity of the Mass-Spring system in different mechanical cases. All results of these tests are compared to FEM model to evaluate the accuracy of the proposed approach.

To conclude, definition of the stiffness of spring in a Mass-Spring system depends on several parameters: the mesh structure, the mesh size and the loading. Actually, as in the textile domain or in the soft tissue model, the parameters definition is a complex problem. However, applications developed in the article show that the Mass-Spring model allows to obtain fast geometrical deformation close to the physical reality. This characteristic is useful in the preliminary studies in the industrial context.

The approach allows to obtain an approximate stiffness of each spring which composing a Mass-Spring system for a specific material. However, to obtain accurate results, the stiffness should be adapted according to different criteria: topology of meshing and loading. For a rubber material, the magnitude of the stiffness is about $1000N/m$ and for a steel material, the magnitude of the stiffness is about $170E6\ N/m$. Then to obtain more accurate results, the stiffness values should be upgraded for each material for each deformation problem.

For the future works, an approach should be developed to take into account the Poisson's ratio to model the behaviour of the deformed material. Moreover the non-linear behaviour of materials should be take into account to model material such as rubber which stiffness depends on strain. These studies may allow to obtain a deformation model closer to the physical reality.

6 ACKNOWLEDGMENTS

The authors would like to thank the Michelin company, and particularly the R&D-XAO department, for their financial support.

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